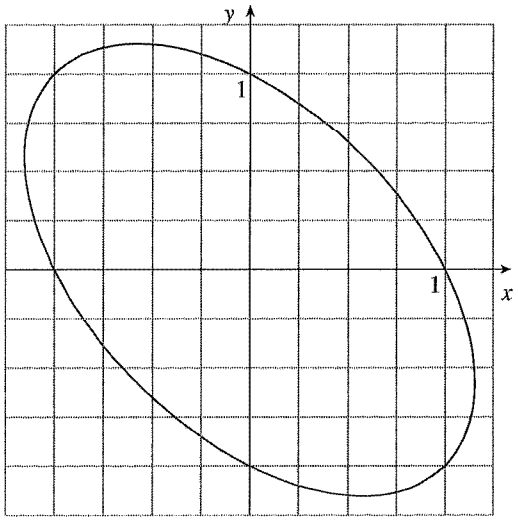
3 Sample Exam Questions

Problems marked with an asterisk (*) are particularly challenging and should be given careful consideration.

1. Consider the equation $x^2 + xy + y^2 = 1$, graphed below.



(a) Find an expression for $\dfrac{dy}{dx}$ in terms of x and y.

(b) Find all points where the tangent line is horizontal.

(c) Find all points where the tangent line is parallel to the line $y = -x$.

2. Let $f(x) = 7\sin(x + \pi) + \cos 2x$.

(a) Compute $f'(x)$, $f''(x)$, $f^{(3)}(x)$, and $f^{(4)}(x)$.

(b) Compute $f^{(13)}(0)$.

3. Assume that $f(x)$ and $g(x)$ are differentiable functions that we know very little about. In fact, assume that all we know of these functions is the following table of data:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	0

(a) Let $h(x) = g(x)\sin x$. What is $h'(0)$?

(b) Let $j(x) = \left[f(x) + x^2\right]^3$. What is $j'(1)$?

4. Let u be a function that is always positive, and suppose that $u'(x) < 0$ for all real numbers x.
   (a) Let $f(x) = [u(x)]^2$. For what values of x will f be increasing?
   (b) Let $g(x) = u(u(x))$. For what values of x will g be increasing?

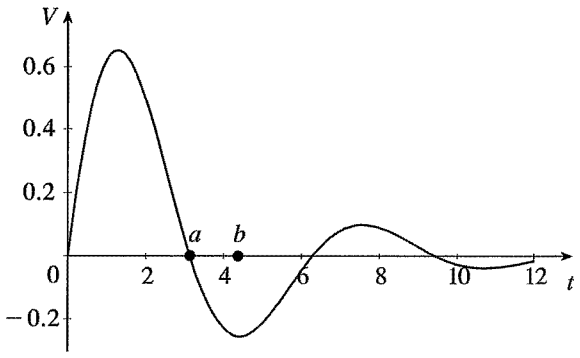
5. Compute derivatives of the following functions.
   (a) $f(x) = e^{2\pi x}$
   (b) $g(x) = x^{2\pi e}$
   (c) $h(x) = (e\pi)^{2x}$
   (d) $l(x) = \pi^{(e^{2x})}$

6. Let $f(x) = -x^3 - 2x^2 + x + 1$ and $g(x) = \ln(x+1) + 1$.
   (a) Find the equation of the line tangent to f at $x = 0$.
   (b) Show that g has the same tangent line as f at $x = 0$.
   (c) Does this tangent line give a better approximation of $f(x)$ or $g(x)$ at $x = 1$? Give reasons for your answer.

*7. Consider the function $h(x) = (1 + \sin \pi x)^{g(x)}$. Suppose $g(1) = 2$ and $g'(1) = -1$. Find $h'(1)$.

8. (a) Find the point on the curve $y = \ln(x^2)$ where the slope of the tangent line is $\frac{2}{3}$.
   (b) Find the x-intercept of the line tangent to the curve at that point.

9. The voltage across a resistor R is given by $V(t) = e^{-0.3t} \sin t$. A graph of V is shown below.



   (a) How fast is the voltage changing after 2 seconds?
   (b) Would you be better off using the linear approximation at $x = a$ to estimate $V(b)$, or using the linear approximation at $x = b$ to estimate $V(a)$? Justify your answer.

10. Let $f(x) = \ln(1 + x^2)$.
   (a) What is $\lim_{x \to \infty} f(x)$?
   (b) What is $\lim_{x \to \infty} f'(x)$?
   (c) Using parts (a) and (b), explain the behavior of the function as x gets large.

11. Consider the curve $x = e^{t-5} \cos t$, $y = e^{t-5} \sin t$, $0 \le t \le 2\pi$.
   (a) Find the values of t where the line tangent to the curve is vertical.
   (b) Find the values of t where the slope of the line tangent to the curve is -1.